

Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use both manipulatives as well as pictorial representations (potentially as part of a **Concrete-Pictorial-Abstract – CPA – approach**) to support their mental and written methods of calculation. As children’s mental methods are strengthened and refined, they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

From Early Years to Year 1:

There are fundamental concepts that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – ‘the ordering of numbers in relation to one another’ – e.g. (1, 2, 3, 4, 5...)

- Cardinality – ‘understanding the value of different numbers’ – e.g. (7 =  17 =  +  14 = 

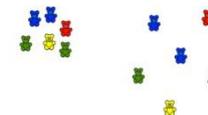
- Equality – ‘seven is the same total as four add three’ – e.g.  = 

- Subitising – ‘instantly recognising the number of objects in a small group, without counting them’ – e.g.  → five

- One-to-one correspondence – e.g.



- Conservation of number – ‘recognising that a value of objects are the same, even if they are laid out differently’ – e.g.

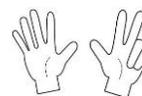


- Concept of zero



$$3 + 0 = 3$$

- Counting on and back from any number – e.g. ‘five add three more totals eight’



- ‘ten take away three totals seven’



The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained. In the 2018 national Key Stage 1 SATs tests, every one of the named mental maths strategies below was assessed, whilst many also featured in a less explicit manner in the Key Stage 2 SATs tests, hence highlighting the need for each method to be taught explicitly. A good knowledge and 'feel' for numbers, is the product of structured practice through progression in relevant practical maths experiences alongside visual representations.

By the end of Year 6, children should be equipped with efficient mental and written calculation methods, which they use fluently. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to move ahead to the next stage. At whatever stage in their learning, and with whatever written method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently with flexibility.

The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value, through experience with practical equipment and visual representations;
- Make use of diagrams (including the bar model) and jottings to help record / reason through stages of thinking when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

Addition:

Mental calculation strategies for addition and subtraction:

All mental calculation strategies need to be taught explicitly using a Concrete – Pictorial – Abstract (CPA) approach in every year group, for example, using decimals in Key Stage 2. The following ideas can be adjusted so that they are accessible to all children. The NCETM, 2015, state that, 'a pupil really understands a mathematical concept, idea or technique if he or she can represent it in a variety of ways.'

Doubles: $8 + 8 = 16$



$8 + 8$ is
connected to
 8×2



Near doubles: $6 + 7 = 13$



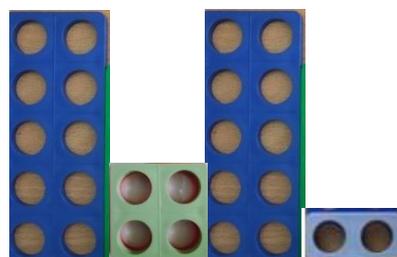
$6 + 7$ is
commutative
with $7 + 6$



Number bonds: $7 + 3 = 10$



Partitioning: $14 + 12 = 26$



Bridging: $7 + 5 = 12$



To begin: $7 + 3 = 10$
Then: $10 + 2 = 12$



Adjusting: $16 + 9 = 25$

To begin: $16 + 10 = 26$

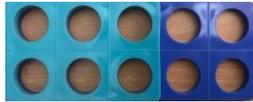


Then: $26 - 1 = 25$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Finding the difference: $10 - 6 = 4$

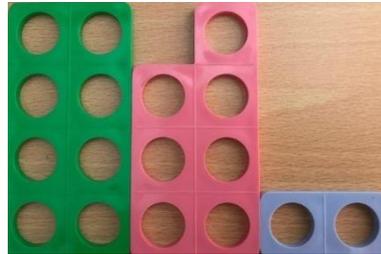


David has 10 sweets, whilst Chloe has six sweets. How many more does David have than Chloe?



Reordering: $8 + 7 + 2 = 17$
e.g. calculating numbers in a different order

To begin: $8 + 2 = 10$
Then: $10 + 7 = 17$



Counting

Mental maths strategies & linked concepts

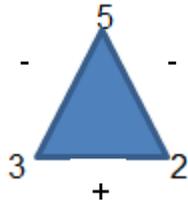
Rapid recall

Written calculation & appropriate models/images to support conceptual understanding

Stage 1:

Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten using a counting stick set up as a number track.

Explicitly teach every mental maths strategy detailed above. Pupils use apparatus to explore addition as the inverse of subtraction.



Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus – i.e. Numicon, tens frames, abaci, etc.

Combining two groups:

- Teachers model how to line up counters/objects on a number track before counting on. This is a precursor to use of a fully numbered number-line.
- Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment.



$3 + 2 = 5$

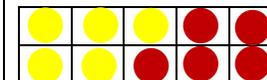
'Three plus two is the same as five'

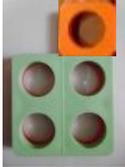
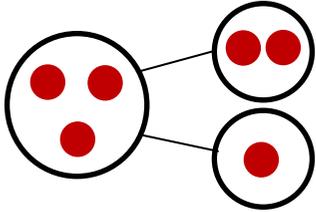
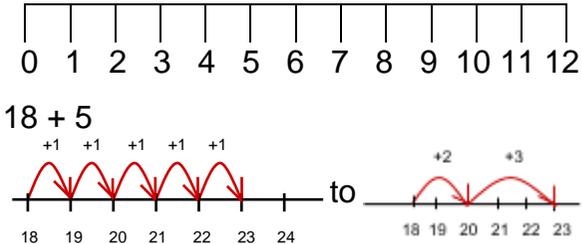
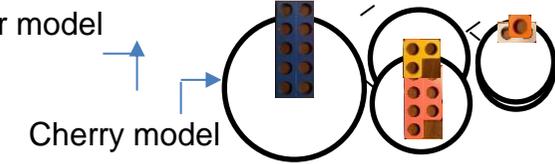
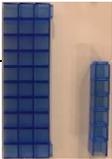
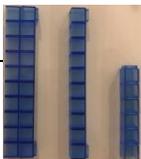


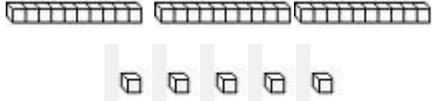
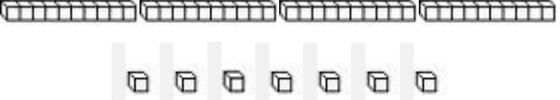
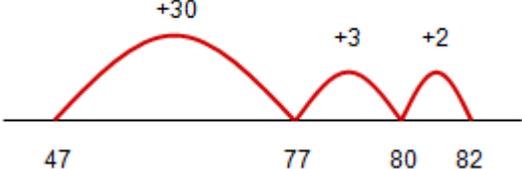
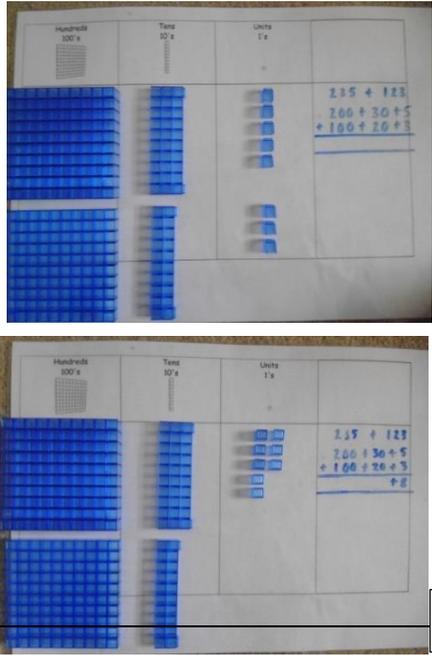
'Eight add two more makes ten'

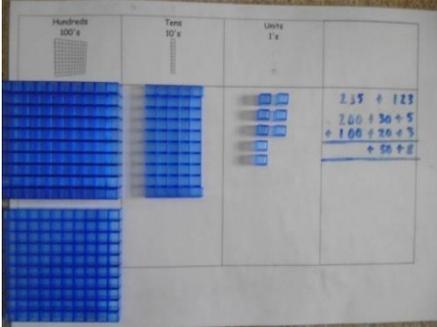
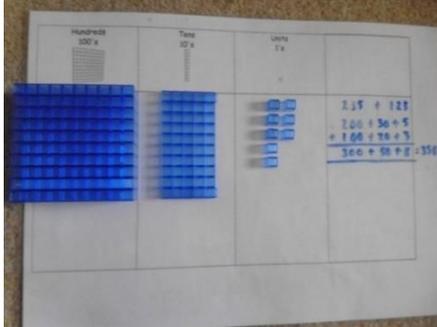
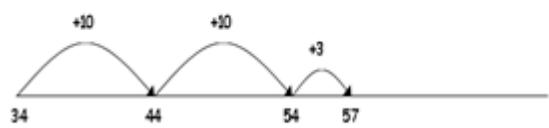


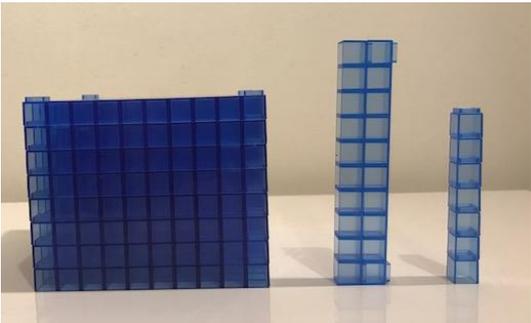
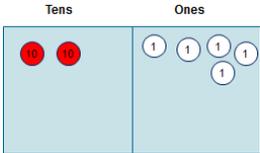
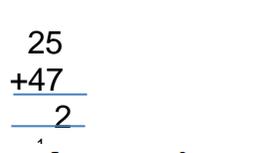
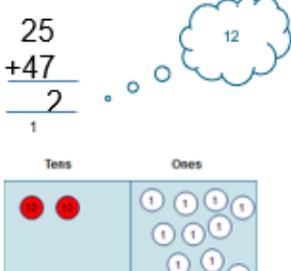
'Four add one more is the same as five'



		<p>'Four add one is the same as five'</p> 		<p>Whole / part-whole model:</p> <ul style="list-style-type: none"> The concept of a whole / part-whole model is introduced. 	<p>Tens frame Bar model</p>  <p>Cherry model</p>
<p>Stage 2:</p>	<p>Continue practising above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero using a counting stick set up as a number line. Count in tens from any number – link to coins in a piggy bank as well as a number square.</p>	<p><u>Explicitly teach every mental maths strategy detailed above.</u> Round numbers to the nearest 10, for example, by illustrating on a number line that is drawn on a folded strip of paper.</p> 	<p>Recall addition facts for all numbers to 20.</p>	<p>Counting on from the largest number:</p> <ul style="list-style-type: none"> Children begin to use number lines to support their own calculations, initially counting on from the largest number in ones before beginning to work more efficiently. <p>Reordering calculations to apply use of mental maths strategies:</p> <ul style="list-style-type: none"> Children reorder 'strings' of numbers to apply their understanding of mental maths strategies, including doubles and number bonds, e.g. $6 + 7 + 4$ reordered to $6 + 4 = 10$ and then $10 + 7 = 17$. Jottings are used to help keep track of thinking. <p>Whole / part-whole model:</p> <ul style="list-style-type: none"> The concept of a whole / part-whole model is reinforced and extended. 	<p>Number line with all numbers labelled</p>  <p>$18 + 5$</p>  <p>Questions such as: 'How might I rearrange these to find the total?' are asked.</p>  <p>Bar model</p>  <p>Cherry model</p>  <p>=</p> 
<p>Stage 3:</p>	<p>Continue practising</p>	<p>Reinforce partitioning and</p>	<p>Connect pairs</p>	<p>Expanded horizontal addition:</p> <ul style="list-style-type: none"> Teachers model how numbers 	

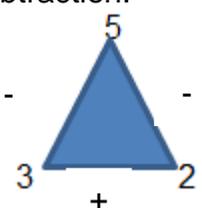
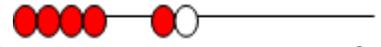
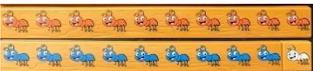
	<p>above skills. Count forward and backwards from 0 in multiples of 4, 8, 50 and 100. Count on 10 or 100 from any two-digit number. Count up and down in tenths. Link to a counting stick as before, whilst deriving number facts.</p>	<p>bridging through multiples of 10, plus adjusting when adding 11 or 9. Use structured apparatus to understand that subtraction undoes addition and link with inverse number operations.</p>	<p>totalling ten to pairs of multiples of 10 totalling 100.</p>  <p>Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. $32 + ? = 100$.</p>	<p>can be partitioned into tens and ones, including different ways, e.g. $36 = 30 + 6$ $36 = 20 + 10 + 6$</p> <ul style="list-style-type: none"> • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line. 	<p>Add...</p>  <p>...and...</p>  <p>By partitioning and recombining</p> $30 + 40 = 70$ $5 + 7 = 12$ $70 + 12 = 82$ <p>$35 + 47$</p> 
<p>Stage 4:</p>	<p>Continue practising previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in</p>	<p>Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to</p>	<p>As above. Use known facts and place value to derive new ones, i.e. 'If I know $8 + 3 = 11$, I also know $0.8 + 0.3 = 1.1$ and $8/100 + 3/100 = 11/100$.' Sums and differences of pairs of</p>	<p>Expanded horizontal method, leading to columnar addition:</p> <ul style="list-style-type: none"> • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete/pictorial materials, e.g. Numicon shapes, Dienes and place-value cards. • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line. 	 <p>a</p> <p>b</p>

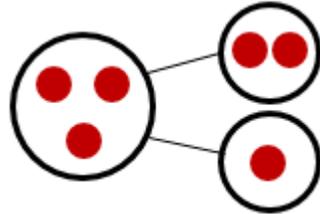
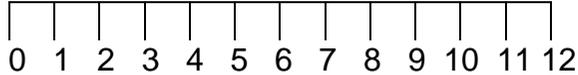
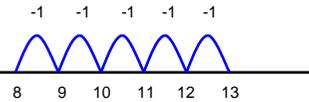
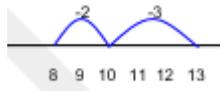
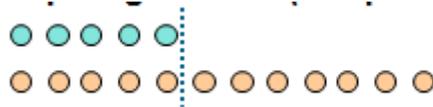
	<p>tenths, hundredths and simple fractions using models and images, plus Dienes / pixie Dienes equipment and a counting stick.</p>	<p>derive new facts and to check accuracy of answers.</p>	<p>multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling one.</p>		<div style="text-align: right;">  <div style="border: 1px solid black; padding: 2px; display: inline-block;">c</div> </div> <div style="text-align: right; margin-top: 10px;">  <div style="border: 1px solid black; padding: 2px; display: inline-block;">d</div> </div> <p>It is important that empty number lines are kept as well as using more formal written calculation methods.</p> <p>Counting on more efficiently:</p> $34 + 23 = 57$ 
<p>Stage 5:</p>	<p>Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count</p>	<p>Use apparatus and knowledge of place value to add decimals, i.e. $3.4 + 2.5 = 5 + 0.9$</p>  <p>Reorder increasingly</p>	<p>Continue to practice previous stage and make links between known facts and addition pairs for fractions,</p>	<p>Expanded vertical method, leading to columnar addition:</p> <ul style="list-style-type: none"> Teachers model a column method that records and explains partial mental methods. There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'... 	<p>Informal columnar:</p> <p>Adding the hundreds first:</p> $\begin{array}{r} 471 \\ + 356 \\ \hline 700 \\ 120 \\ \hline 827 \end{array}$ <p>Adding the ones first:</p> $\begin{array}{r} 471 \\ + 356 \end{array}$

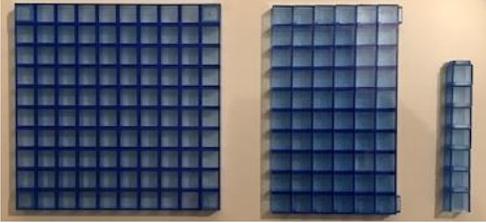
	<p>forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.</p>	<p>complex calculations, i.e. $1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8$ Compensating – i.e. $405 + 399 \rightarrow$ add 400 and then subtract one.</p>	<p>percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. $6.5 + 2.7$</p>	<p>‘Seven add six equals thirteen.’ ...before recombining numbers. Teachers also model the language of: ‘Four tens add seven tens total eleven tens or 110.’</p> <ul style="list-style-type: none"> Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children’s knowledge of place value is secured, they become ready to approach a formal compact method. 	<div style="text-align: right;"> $\begin{array}{r} 7 \\ 120 \\ \hline 700 \\ \hline 827 \end{array}$ </div> 
<p>Stage 6:</p>	<p>Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.</p>	<p>Bridging through decimals, i.e. $0.8 + 0.35 = 0.8 + 0.2 + 0.15$ using empty number lines. Partitioning using near doubles, i.e. $2.5 + 2.6 = 5 + 0.1$ Reorder decimals, i.e. $4.7 + 5.6 - 0.7$...as... $4.7 - 0.7 + 5.6 = 4 + 5.6$.</p>	<p>Using children’s confident recalling of basic facts to 20/100 and deriving facts using place value, make links between decimals, fractions and percentages. i.e. $1 + \frac{19}{10} + \frac{190}{100} + \frac{1900}{1000}$ Question: What do you notice?</p>	<p>Columnar addition (formal written method):</p> <ul style="list-style-type: none"> The concept of exchange is reinforced through continued use of manipulatives. Teachers model: <ol style="list-style-type: none"> “I have two tens and five ones, which need adding to four tens and seven ones.” “I add five ones to seven ones, which gives me twelve ones.” “I exchange ten of my twelve ones for a ten counter.” “I add my three tens and four tens to make seven tens.” “Altogether, I have seven tens and two ones.” Teachers similarly advance to model the addition of two 3-digit numbers and then go 	<p>Pupils to be encouraged to consider mental strategies first.</p> <p>Formal columnar – using an example with smaller value numbers to exemplify:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} 25 \\ +47 \\ \hline \end{array}$  </div> <div style="text-align: center;"> $\begin{array}{r} 25 \\ +47 \\ \hline 72 \\ \hline 1 \end{array}$  </div> </div> <div style="text-align: center; margin-top: 20px;"> $\begin{array}{r} 25 \\ +47 \\ \hline 72 \\ \hline 1 \end{array}$  </div>

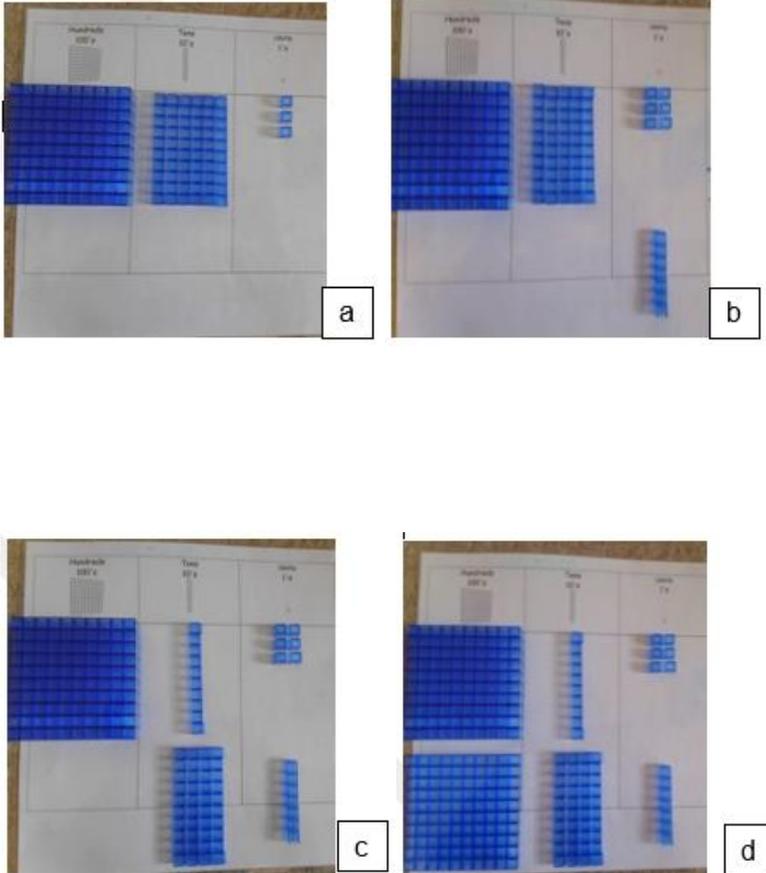
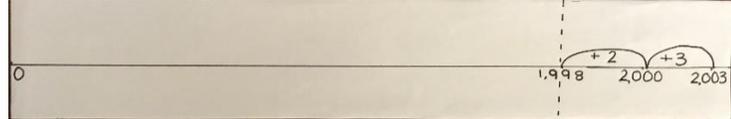
				beyond.	
				$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ 11 \end{array}$	

Subtraction:

	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	<p><u>Explicitly teach every mental maths strategy detailed above.</u></p> <p>Pupils use apparatus to explore addition as the inverse of subtraction:</p>  <p>'One less than five is four'</p> 	<p>Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc.</p> 	<p>Subtraction as taking away from a group:</p> <ul style="list-style-type: none"> Teachers model how to remove counters/objects and count back on a number track. This is a precursor to use of a fully numbered number-line. <p>Whole / part-whole model:</p> <ul style="list-style-type: none"> The concept of a whole / part-whole model is introduced. 	 <p>'Five minus two totals three'</p>  <p>'Six take away two leaves four'</p>  <p>'One less than six is five'</p>  <p>Tens frame</p>  <p>Bar model</p>

					 <p>Cherry model</p>
<p>Stage 2:</p>	<p>Continue practising above skills. Count in steps of 2, 3 and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.</p>	<p><u>Explicitly teach every mental maths strategy detailed above.</u></p>	<p>Recall subtraction (and addition) facts for all numbers to 20.</p>	<p>Taking away:</p> <ul style="list-style-type: none"> Children begin to use number lines to support their own calculations, initially counting back in ones before beginning to work more efficiently. <p>Finding the difference:</p> <ul style="list-style-type: none"> Teachers model how to find the difference when two numbers are relatively 'close together.' 	<p>Number line with all numbers labelled</p>  <p>$13 - 5 = 8$</p>  <p>to</p>  <p>Comparing two sets to find the difference.</p>  <p>Subtraction by partitioning with use of manipulatives and linked with a horizontal expanded written algorithm:</p> $167 - 24 = 143$ 
<p>Stage 3:</p>	<p>Continue practising above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by</p>	<p>Reinforce partitioning and bridging through multiples of 10, plus adjusting when subtracting 11 or 9. Use structured apparatus to understand that</p>	<p>Connect subtractions from ten to subtractions from multiples of 10 totalling 100.</p>	<p>Taking away:</p> <ul style="list-style-type: none"> When teaching children about reduction, highlight the importance of only partitioning one number. 	<p>Subtraction by partitioning with use of manipulatives and linked with a horizontal expanded written algorithm:</p> $167 - 24 = 143$ 

	<p>10 or 100 from any two digit number. Link to counting stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.</p>	<p>subtraction undoes addition and link with inverse number operations.</p>	 <p>Use 10ps in tens frame. Subtract two digit numbers from 100 i.e. $? = 100 - 78$</p>	<p>Finding the difference:</p> <ul style="list-style-type: none"> Children move on to find the difference by making number line comparisons. 	 <p>In either order... To begin: $167 - 20 = 147$</p>  <p>Then: $147 - 4 = 143$</p>  $\begin{array}{r} 100 + 60 + 7 \\ - \quad 20 + 4 \\ \hline 0 + 40 + 3 \end{array}$ <p>Finding the difference on a number line:</p>  <p>Children should note that finding the difference is often the most efficient way of solving a subtraction problem when two numbers are close together. e.g. $61 - 59$</p>
<p>Stage 4:</p>	<p>Continue practising of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using</p>	<p>Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest</p>	<p>As above. Use known facts and place value to derive new ones, i.e. 'If I know $11 - 3 = 8$, I also know $1.1 - 0.3 = 0.8$ and</p>	<p>Taking away:</p> <ul style="list-style-type: none"> Subtraction by partitioning with use of manipulatives, and including transfer / exchange, linked with a horizontal expanded 	<p>$363 - 147 = 216$</p> $\begin{array}{r} 50 \quad 13 \\ 300 + 60 + 3 \\ - 100 + 40 + 7 \\ \hline 200 + 10 + 6 \end{array}$

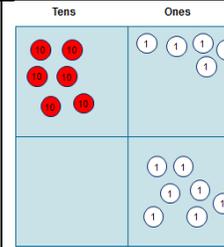
	<p>counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes / Pixie Dienes equipment, counting stick, ITPs.</p>	<p>whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.</p>	<p>8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Subtraction of fractions totalling 1, i.e. $1 - 0.3 = 0.7$</p>	<p>written algorithm in preparation for a future formal column method.</p> <p>Finding the difference:</p> <ul style="list-style-type: none"> Finding the difference continues to be highlighted where the two numbers are close together – using a number line on a strip of paper. 	 
<p>Stage 5:</p>	<p>Count forwards and backwards in steps of powers of 10</p>	<p>Use apparatus and knowledge of place value to subtract decimals, i.e. $3.8 - 2.5 = 1.3$</p>	<p>Continue to practise previous stage and make links</p>	<p>Column method with Dienes:</p> <ul style="list-style-type: none"> Subtraction by partitioning with use of 	$ \begin{array}{r} 51 \\ 363 \\ - 147 \\ \hline 216 \end{array} $

	<p>for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.</p>	<p>Reorder increasingly complex calculations, i.e. $1.7 - 0.5 - 0.7 = 1.7 - 0.7 - 0.5$. Compensating – i.e. $405 - 399 \rightarrow$ subtract 400 and then add 1.</p>	<p>between known facts and addition pairs for fractions, percentages and decimals. Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. $6.5 + 2.7$</p>	<p>manipulatives, and including transfer / exchange, linked with a formal column written algorithm.</p>													
<p>Stage 6:</p>	<p>Continue to practise previous skills. Count forwards and backwards in simple fractions, decimals and percentages.</p>	<p>Bridging through decimals, i.e. $1.5 - 0.8 = 1.5 - 0.5$ then $- 0.3$ using empty number line.</p>	<p>Using children's confident recalling of basic facts to 20/100 and using place value, make links between decimals, fractions and percentages. $19 - 1 =$ $190 - 10 =$ $1900 - 100 =$ $1.9 - 0.1 =$ Question: What do you</p>	<p>Column method with place value counters:</p> <ul style="list-style-type: none"> The concept of transfer / exchange is continued through use of manipulatives. Teachers model: <ol style="list-style-type: none"> "I have seven tens and two ones. I need to subtract four tens and seven ones." "At the moment, I 	<p>Pupils to be encouraged to consider mental strategies first.</p> <p>Formal columnar – using an example with smaller value numbers to exemplify:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $\begin{array}{r} 72 \\ - 47 \\ \hline \end{array}$ <table border="1" style="margin: 0 auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Ones</td> </tr> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </table> </div> <div style="text-align: center;"> $\begin{array}{r} 6\overset{1}{\cancel{7}}2 \\ - 47 \\ \hline \end{array}$ <table border="1" style="margin: 0 auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Ones</td> </tr> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </table> </div> <div style="text-align: center;"> $\begin{array}{r} 6\overset{1}{\cancel{7}}2 \\ - 47 \\ \hline 25 \end{array}$ <table border="1" style="margin: 0 auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Ones</td> </tr> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </table> </div> </div>	Tens	Ones			Tens	Ones			Tens	Ones		
Tens	Ones																
Tens	Ones																
Tens	Ones																

notice?

cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones.”

3. “Now I can take away seven ones from twelve ones, so that I have five ones left.”
4. “I can now subtract four tens from six tens, which leaves me with two tens.”
5. “I recombine two tens and five ones to understand that I am left with twenty-five.”

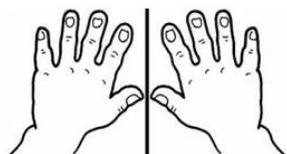
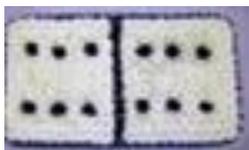


Multiplication:

Mental calculation strategies for multiplication and division:

Doubling and halving:

Double six is 12... Double five is ten...



Double 16 can be calculated by working out...

Double ten → 20

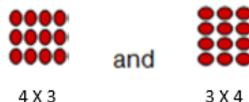
Double six → 12



With links to finding four-times a given value and finding a quarter of a value.

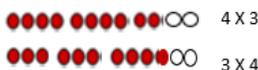
Knowing multiplication and division facts to 12 X 12:

Arrays:



4 X 3 and 3 X 4

Number lines:



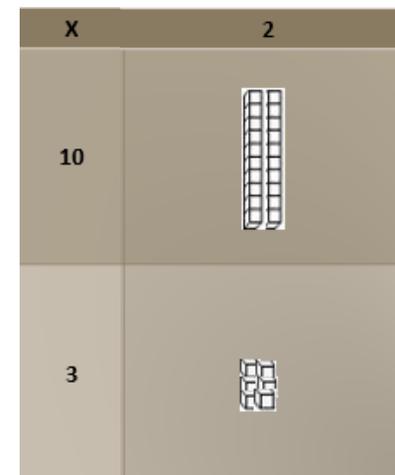
4 X 3 and 3 X 4

Scaling:



Three times longer

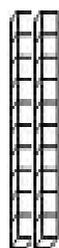
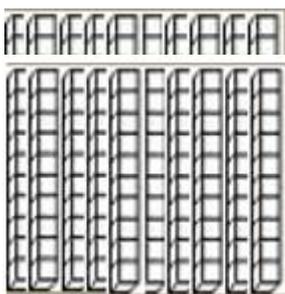
Multiplying a teen number by one-digit number:



Multiplying and dividing by multiples of ten:

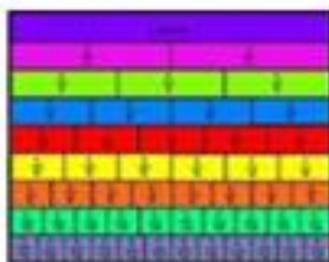
$$20 \times 10 = 200$$

Hundreds	Tens	Ones
	2	0



'Add a place value holder'

Identifying fractions, decimals and percentages:



Milk the maths...



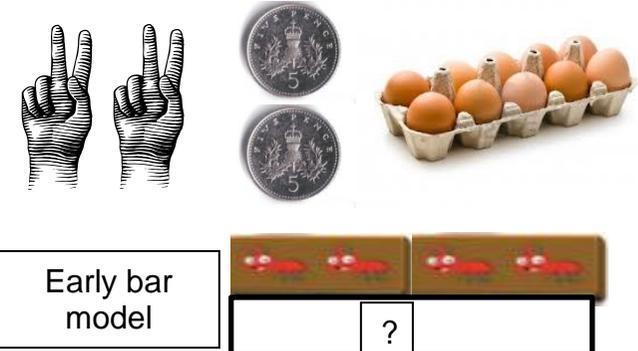
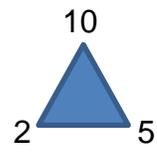
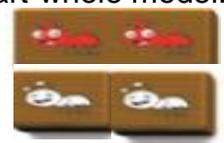
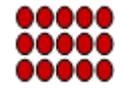
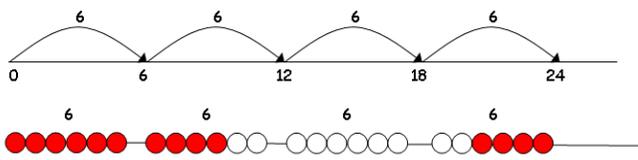
...by allowing children to make connections between number facts.

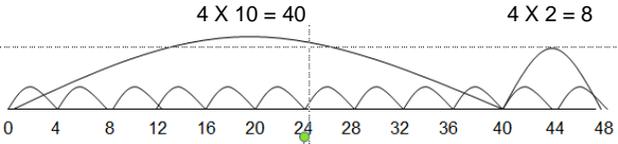
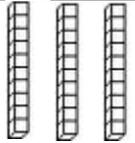
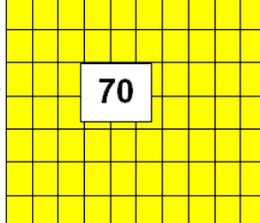
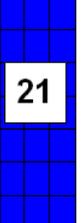
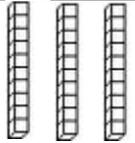
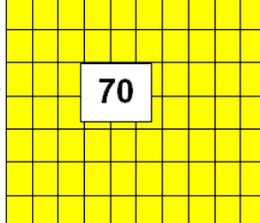
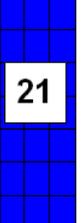
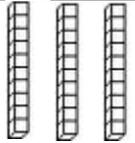
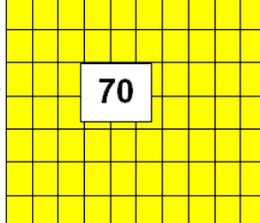
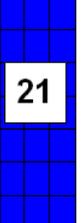
Counting

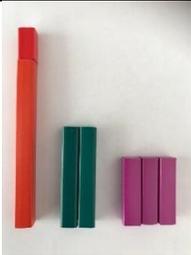
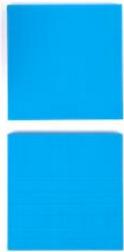
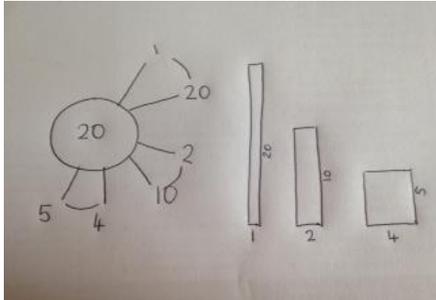
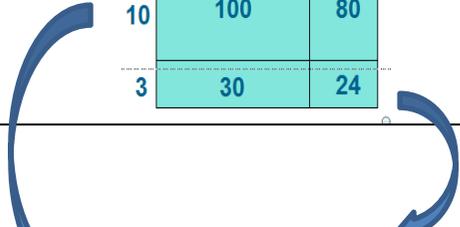
Mental strategies

Rapid recall

Written calculation and appropriate models and images to support conceptual understanding

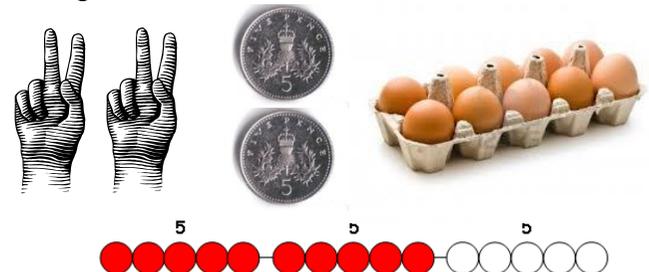
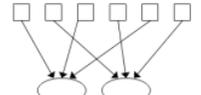
<p>Stage 1:</p>	<p>Count forwards and backwards in 2s, 5s and 10s</p>	<p>Doubling up to six and then ten whilst using related models and images.</p>	<p>Derive/recall doubles up to five and derive/recall halves up to ten.</p>	<p>Developing early conceptual understanding of multiplication (grouping):</p>	<p>Use objects, pictorial representations and arrays to show the concept of multiplication:</p> 
<p>Stage 2:</p>	<p>Count forwards and backwards in 2s, 3s, 5s and 10s from zero.</p>	<p>Begin to understand and use inverse number operations:</p> <div style="text-align: center;">  </div> <p>Stories are used alongside a triad to help children understand links between number operations, e.g. “There are five pencils in two packs, which means that there are ten pencils altogether.”</p> <p>Doubling is reinforced using a whole/part-whole model:</p> 	<p>Derive/recall doubles up to ten and derive/recall halves up to twenty.</p> <p>Recall & use multiplication facts for the 2X, 5X and 10X-tables.</p> <p>Learn what happens when a number is multiplied by zero or one.</p>	<p>Understanding multiplication as repeated addition:</p> <ul style="list-style-type: none"> Investigate multiplication as repeated addition, so that the law of commutativity is understood. Whilst arrays are also modelled explicitly at this stage, it is important to note that they will continue to be a key model at later stages, alongside more formal methods of calculation. 	<p>Arrays:</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> 5×3  </div> <div style="text-align: center;"> <p>and</p> </div> <div style="text-align: center;"> 3×5  </div> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Connect related facts with both array and repeated addition images.</p> </div> <p>Repeated addition on the number line linked with manipulatives:</p> <p>$6 \times 4 = 24$</p>  <p>So: ‘Six multiplied by four’ ...or... ‘Six taken four times.’</p>

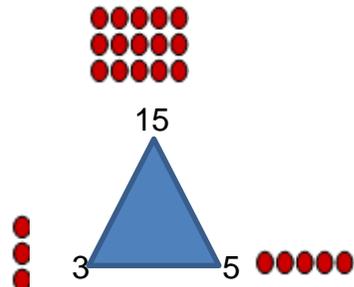
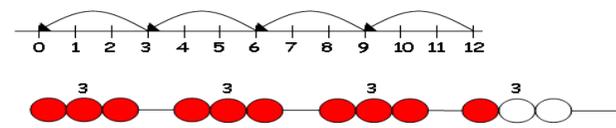
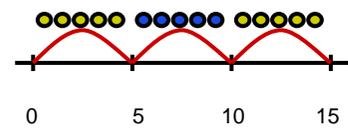
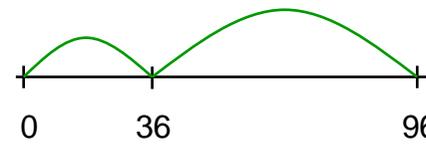
<p>Stage 3:</p>	<p>Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.</p> <p>Count up and down in tenths.</p>	<p>Use doubling to make connections between the 2X, 4X and 8X-tables.</p> <p>Understand that multiplication can be undertaken by partitioning numbers, e.g. $12 \times 4 = 10 \times 4 + 2 \times 4$</p> <p>Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon</p> 	<p>Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.</p>	<p>Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent:</p>	<p>Children use an empty number line to group efficiently:</p> <p>$4 \times 12 = 48$</p>  <p>$3 \times 13 = 39$</p> <table border="1" data-bbox="1460 427 2145 689"> <tr> <td>X</td> <td>10</td> <td>3</td> </tr> <tr> <td>3</td> <td></td> <td></td> </tr> </table> <p>$7 \times 13 = 91$</p> <table border="1" data-bbox="1460 746 1832 1008"> <tr> <td>X</td> <td>10</td> <td>3</td> </tr> <tr> <td>7</td> <td></td> <td></td> </tr> </table>	X	10	3	3			X	10	3	7		
X	10	3															
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<p>Stage 4:</p>	<p>Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.</p> <p>Count up and down in tenths and</p>	<p>Derive factor pairs of numbers using models and images, e.g. Cuisenaire</p> <p>1 and 12 2 and 6 3 and 4</p>	<p>Recall & use multiplication facts for all times-tables up to 12×12.</p>	<p>Relate multiplying a 3 or 2-digit by 1-digit number with arrays towards using long/short multiplication:</p>	<p>Relate multiplying a 3/2-digit by 1-digit number, whilst refining the written notation used.</p> <p>$114 \times 2 = 228$</p> <table border="1" data-bbox="1460 1216 2145 1257"> <tr> <td>X</td> <td>100</td> <td>10</td> <td>4</td> </tr> </table>	X	100	10	4								
X	100	10	4														

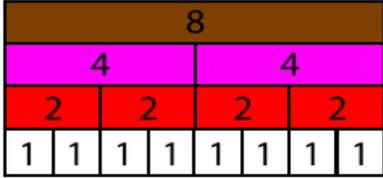
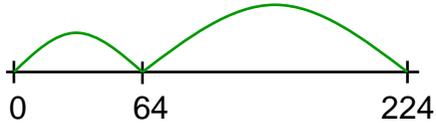
	<p>hundredths.</p>	 <p>Use reordering to multiply three numbers.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Children learn about the associative law: $(9 \times 5) \times 10 = (10 \times 5) \times 9$</p> </div>			<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>2</p>  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>114 X 2 =</p> <table style="margin-left: 20px;"> <tr><td>100 X 2 = 200</td></tr> <tr><td>10 X 2 = 20</td></tr> <tr><td>4 X 2 = 8</td></tr> <tr><td style="border-top: 1px solid black;">= 228</td></tr> </table> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Link with distributive law: $(100 \times 2) + (10 \times 2) + (4 \times 2) = 228$</p> </div> <p>At this stage, the non-statutory guidance in the National Curriculum suggests teaching short multiplication; however, the team feel that an expanded form of calculation (as set out above) is a better lead into long/short multiplication.</p>	100 X 2 = 200	10 X 2 = 20	4 X 2 = 8	= 228								
100 X 2 = 200																	
10 X 2 = 20																	
4 X 2 = 8																	
= 228																	
<p>Stage 5:</p>	<p>Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.</p>	<p>Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.</p> 	<p>Recall & use multiplication facts for all times-tables up to 12 X 12.</p>	<p>Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using long multiplication:</p>	<table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td style="text-align: center;">10</td><td style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">10</td><td style="border: 1px solid black; padding: 5px;">100</td><td style="border: 1px solid black; padding: 5px;">80</td><td></td></tr> <tr><td style="text-align: center;">3</td><td style="border: 1px solid black; padding: 5px;">30</td><td style="border: 1px solid black; padding: 5px;">24</td><td></td></tr> </table> <p style="margin-left: 20px;"> 18 X13 ----- 24 30 80 100 ----- 234 </p>			10	8	10	100	80		3	30	24	
		10	8														
10	100	80															
3	30	24															
<p>Stage 6:</p>	<p>Consolidate all previous counting, including forwards and</p>	<p>Perform mental calculations, including with mixed numbers and operations.</p>	<p>Recall & use multiplication facts for all times-tables up to 12 X 12.</p>	<p>Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using short multiplication:</p>	<table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td style="text-align: center;">10</td><td style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">10</td><td style="border: 1px solid black; padding: 5px;">100</td><td style="border: 1px solid black; padding: 5px;">80</td><td></td></tr> <tr><td style="text-align: center;">3</td><td style="border: 1px solid black; padding: 5px;">30</td><td style="border: 1px solid black; padding: 5px;">24</td><td></td></tr> </table> 			10	8	10	100	80		3	30	24	
		10	8														
10	100	80															
3	30	24															

	backwards in fractions.				$\begin{array}{r} 18 \\ \times 13 \\ \hline 54 \\ 180 \\ \hline 234 \end{array}$ <p>Once children have fully grasped the concept of multiplication alongside manipulatives and an expanded written method, they will be well-placed to progress towards a more compact written algorithm.</p>
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Division:

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten.	<p>Developing early conceptual understanding of division as grouping and sharing:</p> <p>Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing.</p>  <p>“Two children share six pencils between them”</p>  <p>“Six children are asked to get into three equal groups”</p> 

<p>Stage 2:</p>	<p>Count forwards and backwards in 2s, 3s, 5s and 10s from zero.</p>	<p>Begin to understand and use inverse number operations.</p>  <p>Stories are used alongside a triad to help children understand links between number operations, e.g. “15 children are asked to get into three groups and find out that there are five people in each group.”</p>	<p>Derive/recall doubles up to ten and derive/recall halves up to twenty.</p> <p>Recall and use multiplication facts for the 2X, 5X and 10X-tables.</p>	<p>Understanding division as repeated subtraction:</p> <ul style="list-style-type: none"> Investigate division as repeated subtraction. Through teacher modelling, children need to know that division is not commutative. 	<p>Number lines:</p> <p>$12 \div 3 = 4$</p>  <p>$15 \div 5 = 3$</p>  <p>Early bar model</p> 
<p>Stage 3:</p>	<p>Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.</p>	<p>Use doubling to make connections between the 2X, 4X and 8X-tables.</p> <p>Understand that multiplication can be undertaken by partitioning numbers, e.g. $12 \times 4 = 10 \times 4 + 2 \times 4$</p> <p>Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon.</p> 	<p>Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.</p>	<p>Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:</p>	<p>Children use an empty number line to chunk efficiently.</p> <p>$96 \div 6 = 16$</p> <p>$6 \times 6 = 36$ $10 \times 6 = 60$</p>  <p>Conceptual understanding can be provided through use of a bead string to highlight the chunks.</p> 
<p>Stage 4:</p>	<p>Counting forwards and</p>	<p>Derive factor pairs of numbers using models and images, e.g. Cuisenaire.</p>	<p>Recall & use multiplication facts for all</p>	<p>Dividing a 3 or 2-digit by 1-digit number,</p>	<p>Children use an empty number line to chunk efficiently.</p>

	backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.		times-tables up to 12 X 12.	representing this efficiently on a number line, also in relation to long division: <ul style="list-style-type: none"> At this stage, remainders may be present in a practical context. 	$224 \div 8 = 28$ $8 \times 8 = 64$ $20 \times 8 = 160$  $\begin{array}{r} 28 \\ 8 \overline{) 224} \\ - 160 \quad (8 \times 20) \\ \hline 64 \\ - 64 \quad (8 \times 8) \\ \hline 0 \end{array}$...or... $\begin{array}{r} 28 \\ 8 \overline{) 224} \\ 20 \times 8 = 160 \\ \hline 64 \\ 8 \times 8 = 64 \\ \hline 0 \end{array}$
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	Dividing a 4/3/2-digit by 1-digit number, in relation to long division: <ul style="list-style-type: none"> By this stage, there is a statutory requirement that children can use a formal written calculation method, such as long division. Short division may begin to be taught alongside long division, but still with use of visual representations 	As schools have autonomy to decide children's progression in learning between long and short division in Years 5 and 6, the maths team suggest beginning with long division. Remainders should be interpreted in the following ways when long division is used: <ul style="list-style-type: none"> as whole numbers as fractions through rounding in an appropriate way to the context Long division: $415 \div 9 = 46 \text{ and } 1/9$ $\begin{array}{r} 46 \text{ and } 1/9 \\ 9 \overline{) 415} \\ 40 \times 9 = 360 \\ \hline 55 \\ 6 \times 9 = 54 \\ \hline 1 \end{array}$

<p>Stage 6:</p>	<p>Consolidate all previous counting, including forwards and backwards in fractions.</p>	<p>Perform mental calculations, including with mixed numbers and different number operations.</p>	<p>Recall & use multiplication facts for all times-tables up to 12 X 12.</p>	<p>Dividing a 4/3/2-digit by 2/1-digit number, in relation to long and then short division:</p> <ul style="list-style-type: none"> By this stage, there is a statutory requirement that children can use formal written calculation methods, including long and short division. Use of visual representations – like the ones opposite – remain important. 	<p>As schools have autonomy to decide children's progression in learning between long and short division in Years 5 and 6, the maths team suggest moving from long division to short division. Remainders should be interpreted in the following way when short division is used:</p> <ul style="list-style-type: none"> through rounding in an appropriate way to the context <p>Long division: $432 \div 15 = 28 \frac{4}{5}$</p> $\begin{array}{r} 28 \\ 15 \overline{)432} \\ \underline{20} \\ 132 \\ \underline{120} \\ 12 \end{array}$ $\frac{12}{15} = \frac{4}{5}$ <p>Answer: $28 \frac{4}{5}$</p> <p>Short division: $138 \div 6 = 23$</p>
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